Two-way Analysis of Variance
Part 2 + 3

Learning objectives

- Two-way factorial design
- Principles of two-way ANOVA
  - Between-participants design
  - Within-participants (repeated-measures) design (Part 3)
- Read research papers
- Read SPSS's printouts

Research paper

- Effects of alcohol and caffeine on driving ability
- 4 conditions:
  - No alcohol; no caffeine
  - alcohol; no caffeine
  - No alcohol; caffeine
  - Alcohol; caffeine
- 2 x 2 factorial design

Two-way factorial design

- Experimental design in which every level of every factor is paired with every level of every other factor
- 2 x 2 factorial: Each factor has two levels
- 3 x 4 factorial: First factor has three levels and second factor has four levels

Two-way factorial design: Example

- Effects of alcohol and caffeine on driving ability
- 2 factors:
  alcohol (No alcohol, low alcohol, high alcohol)
  caffeine (No caffeine, caffeine)
- 2 x 3 factorial design:
  - No alcohol; no caffeine
  - No alcohol; caffeine
  - Low alcohol; no caffeine
  - Low alcohol; caffeine
  - High alcohol; no caffeine
  - High alcohol; caffeine

Research paper

- Effects of alcohol and caffeine on driving ability
- 2 factors:
  alcohol (No alcohol, alcohol)
  caffeine (No caffeine, caffeine)
- Between-participant design:
  48 participants with one in each condition

Research paper in between-participant design

- **Results:**
  A two-way ANOVA revealed a main effect of alcohol ($F(1, 44) = 339.8$, $p < 0.001$) and of caffeine ($F(1,44) = 515.4$, $p < 0.001$). The interaction between alcohol and caffeine was significant as well ($F(1,44) = 37.8$, $p < 0.001$).

One-way ANOVA

- **Can main effect and interactions be determined by an one-way ANOVA?**
- **Each cell is a level in an One-way ANOVA**

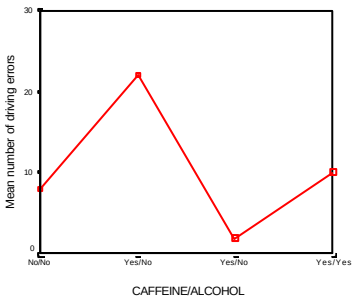
One-way ANOVA

ANOVA

SCORE

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2554.083	3	851.361	297.694	.000
Within Groups	125.833	44	2.860		
Total	2679.917	47			

One-way ANOVA



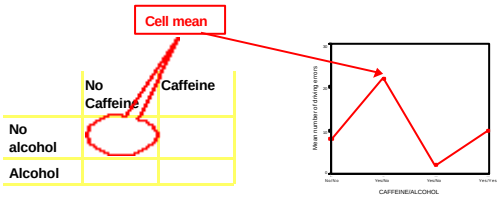
One-way ANOVA

- **Cannot separate the influence of more then one independent variable (main effect)!**
- **Cannot show any interaction between independent variables!**

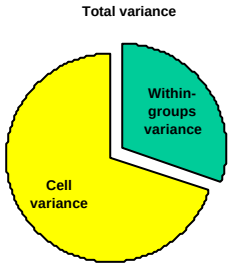
Two-way ANOVA (between-participant design)

Cell

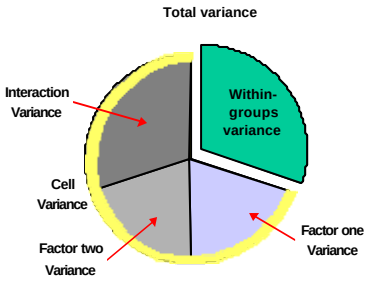
- Any combination of one level of one factor with one level of another factor



Partitioning variance



Partitioning variance



Partitioning variance

- Total variance
- Within-groups (Error) variance
- Variance of cells:
  (between-groups variance in one-way ANOVA)
  - Variance of each factor
  - Variance of interaction between factors

F-statistic in two-way ANOVA

- Broadly:
  Each F is the ratio between
  Each variance
  and
  Within-groups (Error) variance

- P-value given by
  F-distribution: $F(df_{(\text{factor or interaction})}, df_{error})$

Degrees of freedom

- df_{Total} = number of participants - 1
- $df_{factor1}$ = number of factor1 levels - 1
- $df_{factor2}$ = number of factor2 levels – 1
- $df_{interaction} = df_{factor1} * df_{factor2}$
- $df_{error} = df_{Total} - df_{factor1} - df_{factor2} - df_{interaction}$

SPSS output

Main effect

Tests of Between-Subjects Effects

Dependent Variable: SCORE

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	2554.083[a]	3	851.361	297.694	.000
Intercept	5250.083	1	5250.083	1835.791	.000
CAFFEINE	972.000	1	972.000	339.878	.000
ALCOHOL	1474.083	1	1474.083	515.441	.000
CAFFEINE * ALCOHOL	108.000	1	108.000	37.764	.000
Error	125.833	44	2.860		
Total	7930.000	48			
Corrected Total	2679.917	47			

a. R Squared = .953 (Adjusted R Squared = .950)

Interaction

df_{error}

Research paper

- **Results:**
  A two-way ANOVA revealed a main effect of alcohol ($F(1, 44) = 339.8$, $p < 0.001$) and of caffeine ($F(1,44) = 515.4$, $p < 0.001$). The interaction between alcohol and caffeine was significant as well ($F(1,44) = 37.8$, $p < 0.001$).

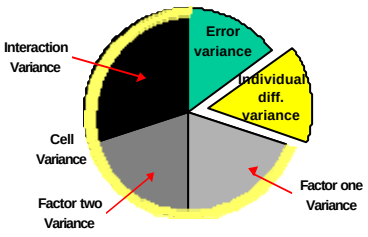
Experiment

- **Effects of alcohol and caffeine on driving ability**
- **2 factors:**
  alcohol (No alcohol, low alcohol, high alcohol)
  caffeine (No caffeine, caffeine)
- **2 x 3 factorial design:**
  - **No alcohol; no caffeine**
  - **No alcohol; caffeine**
  - **Low alcohol; no caffeine**
  - **Low alcohol; caffeine**
  - **High alcohol; no caffeine**
  - **High alcohol; caffeine**

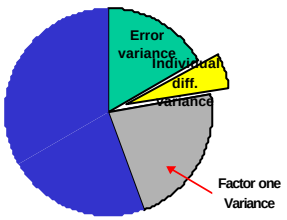
Two-way Analysis of Variance
Part 3

Within-participants design

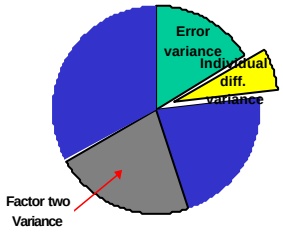
Partitioning variance



Partitioning variance for factor one



Partitioning variance for factor two



F-statistic in two-way ANOVA

- **Broadly:**
  Each F is the ratio between
  Each variance (factor or interaction)
  and
  Error variance without individual differences (factor or interaction)

- **P-value given by**
  F-distribution: $F(df_{(\text{factor or interaction})}, df_{\text{error (factor or interaction)}})$

SPSS output

Main effect of caffeine

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
CAFFEINE	Sphericity Assumed	972.000	1	972.000	577.946	.000
	Greenhouse-Geisser	972.000	1.000	972.000	577.946	.000
	Huynh-Feldt	972.000	1.000	972.000	577.946	.000
	Lower-bound	972.000	1.000	972.000	577.946	.000
Error(CAFFEINE)	Sphericity Assumed	18.500	11	1.682		
	Greenhouse-Geisser	18.500	11.000	1.682		
	Huynh-Feldt	18.500	11.000	1.682		
	Lower-bound	18.500	11.000	1.682		

Error variance without individual differences for caffeine

SPSS output

Interaction between caffeine and alcohol

Error(ALCOHOL)	Sphericity Assumed	23.417	11	2.129		
	Greenhouse-Geisser	23.417	11.000	2.129		
	Huynh-Feldt	23.417	11.000	2.129		
	Lower-bound	23.417	11.000	2.129		
CAFFEINE * ALCOHOL	Sphericity Assumed	108.000	1	108.000	52.800	.000
	Greenhouse-Geisser	108.000	1.000	108.000	52.800	.000
	Huynh-Feldt	108.000	1.000	108.000	52.800	.000
	Lower-bound	108.000	1.000	108.000	52.800	.000
Error(CAFFEINE*ALCOHOL)	Sphericity Assumed	22.500	11	2.045		
	Greenhouse-Geisser	22.500	11.000	2.045		
	Huynh-Feldt	22.500	11.000	2.045		
	Lower-bound	22.500	11.000	2.045		

Error variance without individual differences for interaction

Counterbalancing

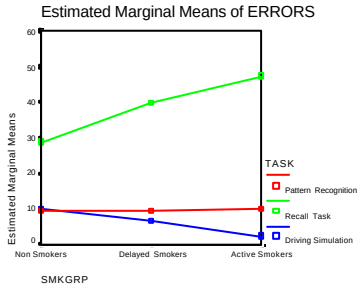
- **Example**
  - **Participant: No alcohol/no caffeine; Alcohol/no caffeine; No alcohol/caffeine; Alcohol/caffeine**
  - **Participant: alcohol/caffeine; No alcohol/no caffeine; No alcohol/caffeine; Alcohol/no caffeine**
  - **Participant: Alcohol/no caffeine; alcohol/caffeine; No alcohol/no caffeine; No alcohol/caffeine; Alcohol/no caffeine**
  - **Participant: ……**

- **24 Combinations**

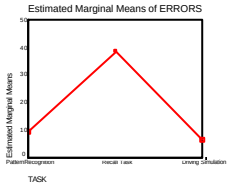
Experiment

- **Effects of smoking on performance**
- **between-participant design (135 participants)**
- **3 Tasks:**
  - **Pattern recognition task**
  - **Recall task**
  - **Driving simulation**
- **3 participant groups:**
  - **Smokers**
  - **Delayed smokers**
  - **Nonsmokers**
- **3x3 factorial design**

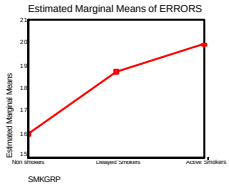
Result: Interaction



Result: Main effects



Task



Smokers

Result

Tests of Between-Subjects Effects

Dependent Variable: ERRORS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	31744.726[a]	8	3968.091	36.798	.000
Intercept	45009.074	1	45009.074	417.389	.000
TASK	28661.526	2	14330.763	132.895	.000
SMKGRP	354.548	2	177.274	1.644	.197
TASK * SMKGRP	2728.652	4	682.163	6.326	.000
Error	13587.200	126	107.835		
Total	90341.000	135			
Corrected Total	45331.926	134			

a. R Squared = .700 (Adjusted R Squared = .681)

Result

- **Main effects:**
  - **Task: F(2,126) = 132.90; p < 0.001**
  - **Groups: F(2,126) = 1.64; p = 0.197**
- **Interaction:**
  - **F(4,126) = 6.33; p < 0.001**

Simple effects

ANOVA

ERRORS

TASK		Sum of Squares	df	Mean Square	F	Sig.
Pattern Recognition 1	Between Groups	2.178	2	1.089	.051	.950
	Within Groups	894.133	42	21.289		
	Total	896.311	44			
Recall Task 2	Between Groups	2643.378	2	1321.689	4.744	.014
	Within Groups	1700.400	42	278.581		
	Total	4343.778	44			
Driving Simulation 3	Between Groups	437.644	2	218.822	9.258	.000
	Within Groups	992.667	42	23.635		
	Total	1430.311	44			

Simple effects

- **Driving simulation: F(2,42)= 0.051; p = 0.950**
- **Pattern Recognition: F(2,42) = 4.744; p = 0.014**
- **Recall Task: F(2,42) = 9.258; p < 0.001**